Gravity-Gradient Stabilization for a Spinning Satellite with Despun Line of Sight

ALFRED S. GUTMAN*
Aerospace Corporation, El Segundo, Calif.

A spin-stabilized cylindrical satellite is suspended either magnetically or electrostatically from a gravity-gradient-stabilized reflector that directs a pencil beam of radiation emanating axially from the satellite toward the earth. Solutions for the motion of a gravity-gradient boom with reference to the local vertical are obtained. The dynamics of the satellite combining spin stabilization with gravity-gradient stabilization result in a stable system with a fixed nutation frequency that can be made high by design. Vibrations in the roll and yaw axes are easily damped by a nutation damper in the spinning cylinder. Damping of the pitch vibration of the reflector can be obtained with a single, cold-gas, pulsed jet that also can be used to prevent angular velocity decay of the spinning cylinder and to control the orbital period for station-keeping. The system described is basically passive as far as the control torques are concerned; however, active means are used to damp pitch vibrations of the nonspinning reflector. Thus, this system can achieve high accuracy of stabilization, in addition to the advantages of high reliability.

Nomenclature

$\mathbf{H}$ = angular momentum vector of spinning portion of satellite
H_x, H_y, H_z, H_v = components of $\mathbf{H}$ in x, y, z, and vertical directions
$\mathbf{M}$ = moment vector of gravity-gradient torque
M_x, M_y, M_v = components of $\mathbf{M}$ in x, y, and vertical directions
$\boldsymbol{\omega}$ = angular velocity vector of spinning portion in inertial space
$\omega_x, \omega_y, \omega_z$ = components of $\boldsymbol{\omega}$ in x, y, and z axes
A = vibration amplitude of boom of unit length
I_x, I_y, I_z = moments of inertia of spinning portion referred to x, y, and z axes
J_x, J_y, J_z = moments of inertia of stationary portion referred to x, y, and z axes
K = integration constant
L = length of boom
L_1 = distance of center of mass of the satellite from the center of rotation of the cylinder
Q = $2\omega_0^2 J_y$
T_x = disturbing torque about x axis
f_p = natural frequency of the stationary part about pitch axis
m = mass of entire satellite
t = time
u = horizontal distance of a boom of unit length from the vertical
x = principal axis (spin axis)
y = axis perpendicular to x and z axes
z = axis in line with boom
x' = horizontal axis normal to orbital plane
y' = horizontal axis in orbital plane
z' = vertical or v axis
Ω = angular vibration frequency of the boom tip in horizontal plane
$\alpha_{x'}, \alpha_{y'}, \alpha_{z'}$ = angles between boom and x', y', and z' axes
β = angle between boom and horizontal plane through spin axis
β' = angle between boom and y' axis projected into orbital plane ($y'z'$ plane)
δ = angle between spin axis and vertical in orbital plane
δ' = angle between boom and x' axis projected into $x'z'$ plane
ϵ_p = pitch angle
ϵ_1 = horizontal plane normal to orbital plane
ϵ_2 = plane through tip of $\mathbf{H}$ parallel to equatorial plane of satellite
φ = angle of rotation about x axis
ψ = angle of rotation about vertical axis (between y and y')
ω_0 = orbital angular velocity of satellite
ω_s = angular velocity of spinning portion referred to stationary portion

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* Member of Technical Staff, Systems Research and Planning Division.

Introduction

SOME satellites have sensors that must point in definite directions. Rather than pointing the sensors or antennas, the attitude of the entire satellite can be stabilized in such a manner that it is possible to use sensors that are fixed with respect to the satellite. However, attitude control systems, as presently used, are a frequent cause of failures of satellites requiring such control. Table 1 summarizes characteristics of present systems. Passive, gravity-gradient systems are reliable but have limited torque capabilities that decrease with the cube of the distance from the center of the earth. On the other hand, active control systems have limitations in reliability. Furthermore, for the cold-gas system,

Table 1 Summary of present satellite attitude stabilization systems

	System		
Characteristics	Passive, gravity-gradient, elastic hysteresis damping	Active, cold-gas, limit cycle	Semiactive, gravity-gradient, control-moment gyroscopes
Acquisition	Slow	Fast	Slow
Accuracy	Poor	Good	Fair
High-altitude performance	Unknown	Good	Unknown
Torque capability	Small	Good	Small
Reliability	Excellent	Poor	(Calculated 61% for 90 days)

the gas storage requirement places a limitation on system life. However, continuous active control systems employing ion engines may become attractive if station-keeping is required. It is the author's opinion that semiactive designs can provide a good compromise between reliability and accuracy.

It is possible to design satellites specifically for maintaining a desired attitude.[1] The gravity-gradient attitude control system could be implemented by designing a mass distribution into the satellite so that passive gravity-gradient stabilization becomes possible without attaching special devices to the satellite, but in practical applications it becomes more convenient to add booms to the satellites. The following discussion will present attitude stabilization methods requiring special satellite designs.

Spin stabilization is based on the principle of the conservation of angular momentum. The satellite is designed essentially as a cylindrical body with a high moment of inertia. The body is spun with the axis of the cylinder normal to the orbital plane. This spin axis has the tendency to stay normal to the orbital plane, because the angular momentum vector of a spinning body is invariant in inertial space, provided that there are no torques acting on the spinning body. Disturbing torques (e.g., from solar radiation pressure or magnetic fields) can cause a precession of the spin axis. In many missions of short duration, this drift of the spin axis may be within acceptable limits of desired attitude stability. Nevertheless, drift of the spin axis is a real problem for long-term missions; the designer may provide torquers, such as cold-gas jets, to precess the spin axis back to the desired orientation. This, of course, is a complication of the basic simple concept of spin stabilization that decreases system reliability.

Use of a directional sensing device on a spinning satellite also has its problems. The only direction in which the sensor can be pointed without motion is in the direction of the spin axis. Since this is necessarily normal to the orbital plane, it is of little practical use for systems that require sensors pointing toward the earth. Such sensors require a rotation opposite to the satellite rotation at the same rate as the satellite in order to point continuously toward the earth. In the Hughes Syncom II satellite, such counterrotation of the antenna beam is accomplished electronically by means of a steerable-phased array of antenna elements arranged on the circumference of the cylindrical rotating satellite.

With respect to scanning, a proposal has been made to use the rotating motion of the spinning satellite by placing a directional sensor pointing outward from the axis of the cylinder. Such a sensor will trace a linear scan on the earth from horizon to horizon in the orbital plane. Since the sensor observes the earth only during a short part of the time it takes to make one revolution, the scan efficiency of this scheme is poor. The higher the altitude, the poorer it becomes. At high altitudes, which are desirable in order to obtain large horizon-to-horizon coverage of the earth, the optical scan efficiency is low. A spinning satellite at low altitude with a radial telescope sees the earth at the most 50% of one revolution and during the remaining 50% of the time looks into empty space. At synchronous altitude the scan efficiency is reduced to about 5%, and for 95% of the remaining time the telescope looks into empty space. The radial scanning telescope, the axial scanning telescope, and the geometry for scan efficiency are shown diagrammatically in Fig. 1.

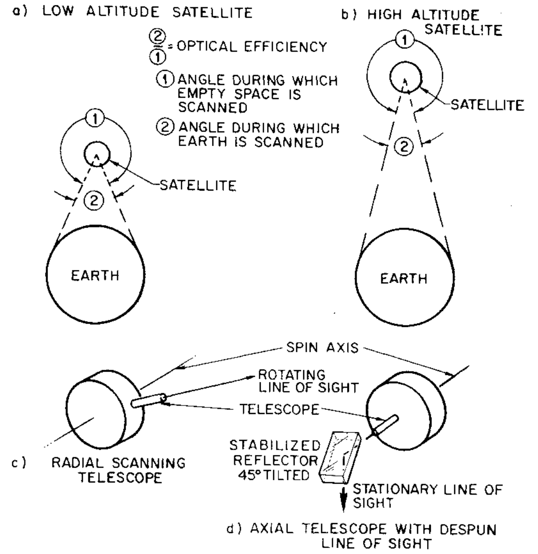

Fig. 1 Methods of mounting telescopes on spinning satellites.

Combined Spin/Gravity-Gradient System

The concept presented herein combines spin stabilization with gravity-gradient stabilization. Figure 2 shows the embodiment of this idea for use in a communications satellite. The satellite instrumentation is housed in a spinning cylinder (spin axis is horizontal in sketch). The paraboloid antenna is coaxial with the cylinder. The spinning cylinder is supported electrostatically inside the gravity-gradient-stabilized mirror assembly that is equipped with a boom to keep the mirror assembly pointed toward the earth. The 45° mirror reflects the radiation from the antenna dish toward the earth. Electrostatic suspension systems of the type required for the cylinder are built by the Minneapolis-Honeywell Regulator Company, Minneapolis, Minn.[2] (An alternative suspension system could be based on the repelling forces experienced by a diamagnetic material in a magnetic field. Such a suspension has been tested experimentally by the General Electric Company, Spacecraft Department, Valley Forge, Pa.[2])

The gravity-gradient-stabilized assembly has the following two purposes: 1) to supply torques to the spin-stabilized

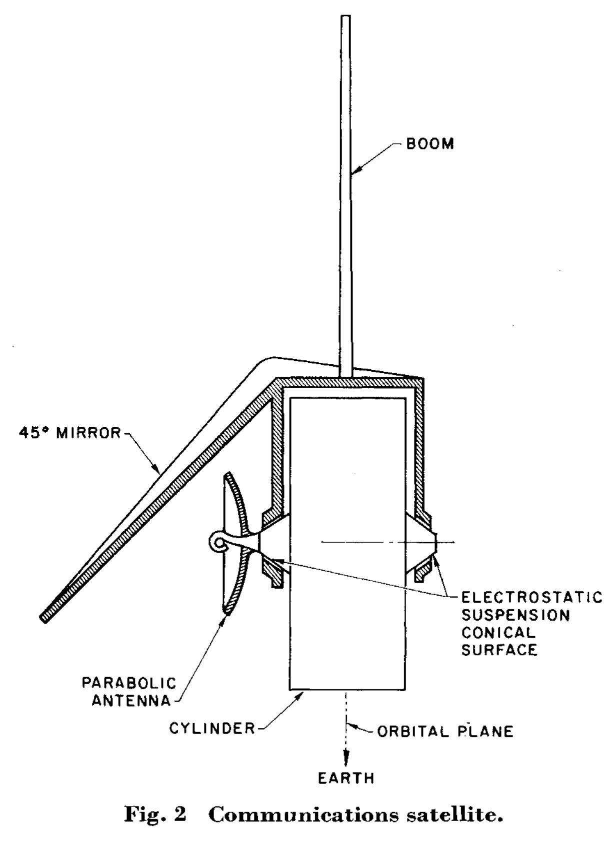

Fig. 2 Communications satellite.

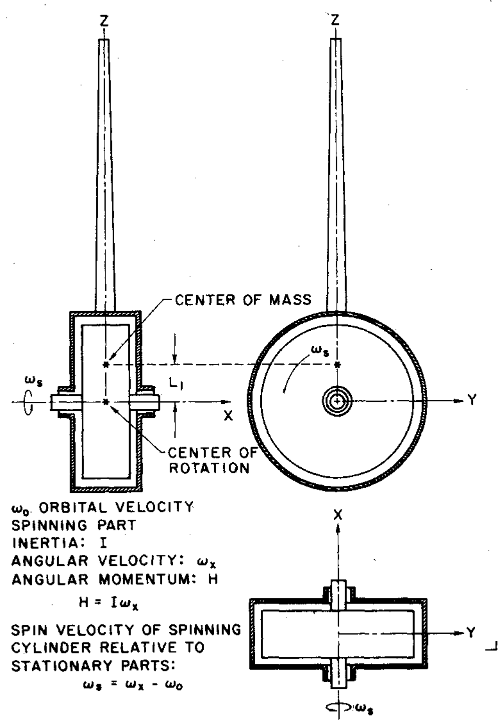

Fig. 3 Dynamic model of satellite.

satellite, so that its spin axis is always kept normal to the orbital plane of the satellite; and 2) to direct beams of radiation emerging parallel to the spin axis of the satellite toward the earth. These two functions are achieved by natural forces without the use of any artificial control system.

At the time of launch the cylindrical satellite rotor is solidly connected to the mirror assembly. After injection into orbit, the electrostatic bearings are energized; then the solid connection between rotor and mirror assembly is broken either by an explosive charge or (in an alternative design) by sublimation of the connecting tissue into the vacuum. Thereafter, the satellite is spun. The injection attitude control system functions during the spin-up; thereafter, it can be turned off. The boom, which deviates from the vertical by an angle smaller than 90°, experiences a restoring torque because of the gravity gradient. This moment vector has a component in line with the spin axis and another component normal to the spin axis. The former acts only on the stationary part of the satellite and does not in any way affect the spinning portion. The other component normal to the spin axis causes a precession of this axis. As a result of the inertia of the satellite, the erecting moment causes a vibration that moves the tip of the boom in a two-dimensional spherical surface. For small vibration amplitudes, we can consider the vibration to be composed of 1) a vibration in the equatorial plane of the satellite (pitch mode) and 2) a vibration in a plane going through the boom and the spin axis of the satellite (roll mode). For the satellite that is vibrating around its nominal stabilized position, these planes are vertical. From Fig. 3 it can be seen that the vibration in the equatorial plane is not coupled to the satellite's spin angular momentum. The period of vibration is the same order of magnitude as the orbital period of the satellite.[3] The vibration in the roll plane is coupled to the satellite's spin angular momentum and has a high fixed frequency (as will be shown later). In addition, the entire satellite assembly can rotate about a vertical axis (yaw mode); this vibration is coupled to the roll mode by the angular momentum of the spinning part of the satellite.

The vibration modes that are coupled to the angular momentum of the spinning satellite contain high fixed frequencies (nutations) that are easily damped by a tuned vibration absorber (nutation damper). The pitch mode, however, is at a low frequency (in the order of magnitude of the orbital rate) and has the problems of damping common to gravity-gradient attitude control systems. Therefore, active pitch damping will be provided by a radially thrusting cold-gas pulsed jet mounted on the spinning assembly, for which the center of rotation is below the center of mass of the satellite as a whole. When the jet is on, the resulting torque reverses with every half revolution of the spinning cylinder. Thus the jet, if timed properly, can also be used to damp the pitch oscillations of the boom. In order to time the jet properly for this purpose, either a pitch angular velocity sensor or a pitch angle detector is required; the latter is used in the present system. A flexible (deflectable) metallic vane in the jet stream provides a tangential component of thrust to compensate for any decay of the spin velocity due to eddy currents or other causes (Fig. 4). If the jet is used for damping of pitch oscillations, its torque should be smaller than the gravity-gradient torque of the boom. As a result of the fact that the spin angular velocity is much higher than the pitch natural frequency, a continuous use of the jet does not excite pitch oscillations.

The jet could also be used for station-keeping, because it can move the satellite anywhere within the equatorial plane of the satellite (the orbital plane). If, for instance, an increase in orbital velocity is required, the jet is switched on when it points backward and switched off when it points forward.

If a leak in the jet should develop, the net effect of this leak on attitude or station-keeping is zero because of the rotation of the jet with the spinning cylinder. For proper operation of the jet, a pitch-attitude sensor in the satellite is required in addition to an orbit period sensor that may be on the ground and have connection to the satellite by a command and control link.

Dynamic Analysis

For the perfectly stabilized satellite (Fig. 3) in a circular orbit, the x axis passes through the spin axis of the satellite, and the z axis passes through the boom. The moments of inertia referred to the coordinate axes are I_x, I_y, and I_z for the spinning portion and J_x, J_y, and J_z for the stationary portion. Cylindrical symmetry is assumed. Therefore, $I_y = I_z$, $J_y = J_x$, and, because of the design of the satellite, $J_z \ll J_y$.

In examining a satellite that is perfectly attitude-stabilized in a circular polar or equatorial orbit, it is found that 1) the z axis is vertical; 2) the angular velocity about the x axis of the stationary part is equal to the orbital angular velocity ω_0; and 3) the angular velocity in inertial space of the spinning part is $\omega_x = \omega_s + \omega_0$, where ω_s is the angular velocity

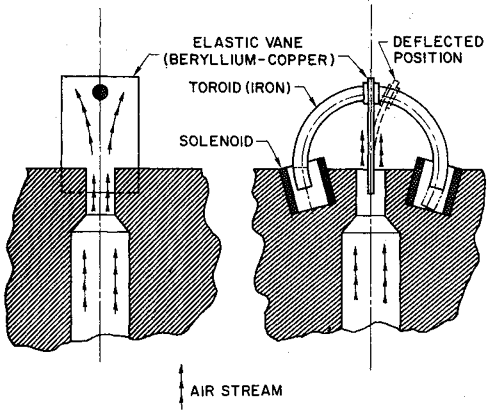

Fig. 4 Jet with deflection vane.

of the spinning part referred to the stationary part. Therefore, the total angular momentum is

$$H = H_x = I_x\omega_x + J_x\omega_0 = I_x\omega_x + J_x(\omega_x - \omega_s) \quad (1)$$

For a perturbed satellite in a circular polar or equatorial orbit, the z axis is no longer identical with the vertical, and therefore a gravity-gradient torque is acting on the boom. Furthermore, the satellite has angular velocities about the z and y axes in addition to the desired spin angular velocity ω_s and orbital angular velocity ω_0. From Fig. 5,

$$\omega_y = d\delta/dt - \omega_0 \sin\delta \sin\psi$$

and

$$\omega_z = (d\psi/dt) \sin\delta + \omega_0 \cos\delta \quad (2)$$

where ϕ, ψ, δ are the Euler angles shown in Fig. 5. When δ, the angle between spin axis and vertical, is near (but not equal to) 90°, and ψ is small,

$$\omega_y \simeq d\delta/dt - \omega_0 \sin\psi \simeq d\delta/dt - \omega_0\psi \quad (3)$$

The angular velocity vector has the following components:

$$\boldsymbol{\omega} = \boldsymbol{\omega}_x + \boldsymbol{\omega}_y + \boldsymbol{\omega}_z \quad (4)$$

Therefore, the angular momentum vector also has components along the y and z axes

$$\mathbf{H} = \mathbf{H}_x + \mathbf{H}_y + \mathbf{H}_z \quad (5)$$

Hence, along with Eq. (1), we have

$$\left.\begin{aligned} H_x &= \omega_x I_x + (\omega_x - \omega_s)J_x \\ H_y &= \omega_y(I_y + J_y) \\ H_z &= \omega_z I_z \end{aligned}\right\} \quad (6)$$

Furthermore, from Fig. 5, $H_v = H_x \cos\delta + H_z \sin\delta$, and therefore

$$H_z = (H_v - H_x \cos\delta)/\sin\delta \quad (7)$$

From Eqs. (3 and 5–7) we have

$$H^2 = [\omega_x I_x + (\omega_x - \omega_s)J_x]^2 + [(I_y + J_y)(d\delta/dt - \omega_0 \sin\psi)]^2 + [(H_v - H_x \cos\delta)/\sin\delta]^2 \quad (8)$$

Because the gravity-gradient torque on the boom depends not only on its deflection from the vertical but also on the direction in the horizontal plane into which it deflects, it is necessary to define the angular relationship between the satellite and the orbital plane. We introduce a new coordinate system; x' is a horizontal axis normal to the orbital plane, y' is a horizontal axis in the orbital plane, and z' is a

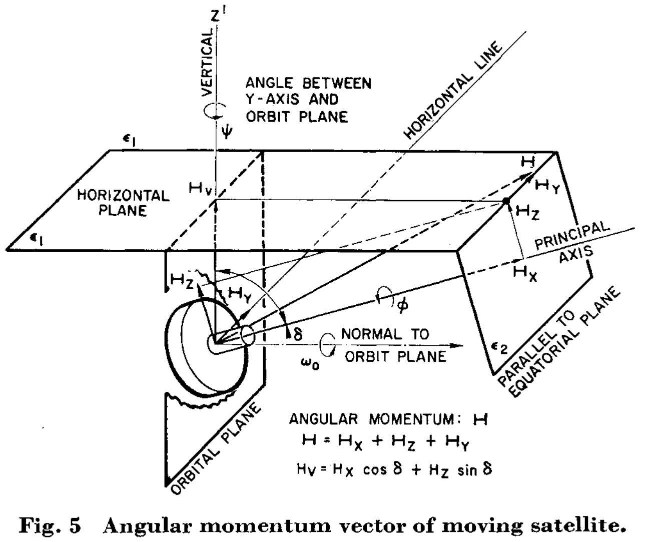

Fig. 5 Angular momentum vector of moving satellite.

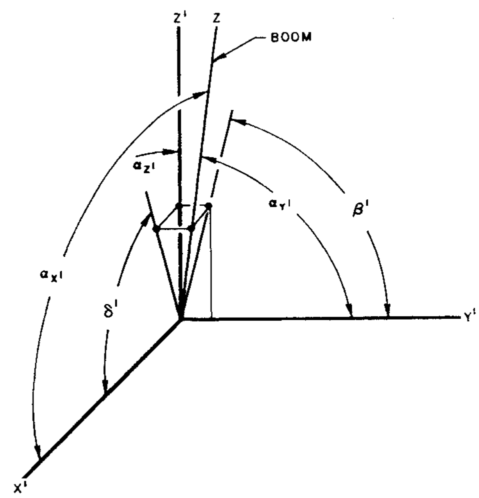

Fig. 6 Angular relationships.

vertical axis. The direction of the vertical is defined by the direction cosines of the angles the vertical makes with the x', y', and z' axes. Referring to Fig. 6, it is desirable to obtain angles δ and β, which correspond to the boom deflection projected into the $x'z'$ and $y'z'$ planes. The result is $\tan\delta' = \cos\alpha_{z'}/\cos\alpha_{x'}$, and $\cot\beta' = \cos\alpha_{y'}/\cos\alpha_{z'}$. If the spin axis of the satellite is assumed to be approximately normal to the orbital plane, $\delta \approx \delta'$. In this case, the equatorial plane of the spinning satellite is approximately coincident with the orbital plane, and $\beta \approx \beta'$.

Qualitative Description of Satellite Motion

The moment vector **M** of the gravity-gradient torque is in a horizontal plane. The component normal to the spin axis M_y acts on the angular momentum of the spinning cylinder and causes a precession of the spin axis. The component parallel to the spin axis M_x causes the stationary part of the satellite to oscillate about the vertical. Suitable means for damping this oscillation must be provided. First, the stability of the satellite will be investigated; then, the action of $\mathbf{M}_x$ and $\mathbf{M}_y$ will be described separately.

The boom, aligned with the z axis, experiences a torque that tries to align z with the vertical. As long as the angle δ is larger than 0°, this restoring torque is positive. Therefore, the satellite is stable for $180° > \delta > 0°$. The restoring torque acting on a rigid attitude-stabilized satellite in the pitch plane with $I_y = I_z$ and $J_z \ll J_y$, and for $\beta \approx 90°$ is

$$M_x = 3\omega_0^2 J_y \beta \quad (9)$$

With a disturbing torque T_x, the error in pitch will be

$$\epsilon_p = T_x/3\omega_0^2 J_y \quad (10)$$

with $J_p = (J_y^2 + J_z^2)^{1/2} \approx J_y$. The linearized uncoupled equation of motion of the satellite in pitch will be

$$J_y\ddot{\epsilon}_p + 3\omega_0^2 J_y \epsilon_p = T_x \quad (11)$$

and the natural frequency of the system is

$$f_p = 2\pi\omega_0(3)^{1/2} \quad (12)$$

With the proposed active damping scheme, the damping torque should be smaller than the restoring torque given in Eq. (9). Equation (12) shows that the vibration period is very long and the angular velocities in pitch are small. This puts exacting requirements on the pitch-attitude sensor, which is necessary for the control of the active damping jets.

Referring to Fig. 5, a horizontal plane ϵ_1 normal to the orbital plane passes through the tip of the angular momentum vector **H**. Furthermore, plane ϵ_2 also passes through the tip of the angular momentum vector **H** and is parallel to the equatorial plane of the spinning satellite. Of course, the tip of $\mathbf{H}_v$ is in ϵ_1, and the tip of $\mathbf{H}_x$ is in ϵ_2. It is necessary to describe the motion of the vector **H** under the influence of the gravity-gradient torque acting on the boom. The moment vector $\mathbf{M}_y$, due to the gravity-gradient torque, is always in a horizontal plane and is perpendicular to the x axis. Hence,

$$\mathbf{M}_x = 0 \quad \text{and} \quad \mathbf{M}_v = 0 \tag{13}$$

The angular momentum vector is

$$d\mathbf{H}/dt = \mathbf{M}_y \tag{14}$$

From Eqs. (13) and (14) it follows that

$$d\mathbf{H}_v/dt = \mathbf{M}_v = 0 \quad \text{and} \quad d\mathbf{H}_x/dt = \mathbf{M}_x = 0 \tag{15}$$

Hence

$$\mathbf{H}_v = \text{const} \quad \text{and} \quad \mathbf{H}_x = \text{const} \tag{16}$$

Therefore, the tip of the **H** vector that must remain in the ϵ_1 and in the ϵ_2 planes will move on the intersection line of the two planes, as shown in Fig. 5. With respect to inertial space, ϵ_1 rotates with orbital angular velocity ω_0 about the x' axis; ϵ_2 rotates with angular velocity ω_z about the z axis.

Analysis of the Motion of the Spin Axis

It is necessary to calculate the motion of the angular momentum vector under the influence of the torques acting on it. The moment vector caused by the gravity gradient is a horizontal vector. Its component M_y normal to the spin axis is required. With the spin axis normal to the orbital plane, $\mathbf{M}_y \approx \mathbf{M}_{y'}$. It can be shown that, in a rigid body that is attitude-stabilized, with $J_z \ll J_y$, this vector is

$$M_y = 2\omega_0{}^2 J_y \sin 2\delta \tag{17}$$

The equation of motion for the spin axis is $d\mathbf{H}/dt = \mathbf{M}$. Because the moment vector **M** is in the y direction, we can take the scalar product

$$\mathbf{H}\cdot d\mathbf{H}/dt = H_y M_y \tag{18}$$

From Eqs. (3) and (6) it follows that

$$H_y = (I_y + J_y)(d\delta/dt - \omega_0 \sin\psi) \tag{19}$$

$$H_z = I_z(d\psi/dt)\sin\delta + I_z\omega_0\cos\delta \tag{20}$$

Combining Eqs. (17–19), we get

$$\mathbf{H}\cdot d\mathbf{H}/dt = (I_y + J_y)(d\delta/dt - \omega_0\sin\psi)Q\sin 2\delta \tag{21}$$

where $Q \equiv 2\omega_0{}^2 J_y$.

Equations (20) and (21), together with Eq. (8), allow us to determine the time-dependent behavior of the roll angle δ and the yaw angle ψ when the initial boundary conditions are known. The coupling of Eqs. (20) and (21) complicates their solution. If $\psi \approx 0$, the solution of Eq. (21) is simplified. It is shown later how the yaw motion of the satellite can be stabilized if the spinning part of the satellite is equipped with a precession and nutation damper. For the present, this result is anticipated, and it is assumed that $\psi = 0$, so that Eq. (21) is

$$\mathbf{H}\cdot d\mathbf{H}/dt = (I_y + J_y)(d\delta/dt)Q\sin 2\delta \tag{22}$$

which can be written as

$$\tfrac{1}{2}dH^2/dt = -\tfrac{1}{2}(I_y + J_y)Q\, d(\cos 2\delta)/dt \tag{23}$$

Integration gives

$$H^2 = -\tfrac{1}{2}(I_y + J_y)Q\cos 2\delta + K_1 \tag{24}$$

where K_1 is an integration constant. Combining Eq. (24) with Eq. (8) and noting that $\psi = 0$, we have

$$K_0 + [(I_y + J_y)d\delta/dt]^2 + [(H_v - H_x\cos\delta)/\sin\delta]^2 + \tfrac{1}{2}Q(I_y + J_y)\cos 2\delta = 0 \tag{25}$$

where

$$K_0 = [I_x\omega_x + J_x(\omega_x - \omega_s)]^2 - K_1 \tag{26}$$

From Eq. (25), the differential equation for δ follows:

$$(I_y + J_y)^2(\sin^2\delta)(d\delta/dt)^2 = [-\tfrac{1}{2}Q(I_y + J_y)\cos 2\delta - K_0]\sin^2\delta - (H_v - H_x\cos\delta)^2 \tag{27}$$

From Figs. 3 and 6 it can be seen that the horizontal distance of the center of mass of the satellite from the local vertical through the center of rotation is $L_1\cos\delta = uL_1$, which defines the quantity u as the horizontal distance of the boom from the vertical for a boom of unit length, since

$$u = \cos\delta \quad \text{and} \quad du/dt = -\sin\delta(d\delta/dt) \tag{28}$$

Combining Eqs. (27) and (28), we get

$$(I_y + J_y)^2(du/dt)^2 = -(uH_x - H_v)^2 - (1 - u^2)[K_0 + \tfrac{1}{2}Q(I_y + J_y)(2u^2 - 1)] \tag{29}$$

For convenience, we introduce

$$F(u) \equiv [(I_y + J_y)du/dt]^2 \tag{30}$$

Then

$$F(u) = -(uH_x - H_v)^2 - (1 - u^2)K_0 - Q(I_y + J_y)(u^2 - \tfrac{1}{2}) \tag{31}$$

Integration of Eq. (30) gives

$$t - t_0 = (I_y + J_y)\int[F(u)]^{-1/2}du \tag{32}$$

which relates the horizontal distance of the boom from the local vertical to time.

In order to obtain meaningful solutions to the foregoing equations for the motion of δ, the *boundary conditions* must be specified. We assume that at time $t_0 = 0$, $d\delta/dt = 0$; hence, with Eq. (2) and $\psi = 0$, $\omega_y = 0$; with Eq. (6), $H_y = 0$; and with Eqs. (28), $du/dt = 0$. Combining $du/dt = 0$ with Eq. (30), we have $F(u_0) = 0$. Furthermore, by definition,

$$\delta = \delta_0 \quad \text{and} \quad \cos\delta_0 = u_0 \tag{33}$$

Combining these with Eqs. (31) and (33) and with $F(u) = 0$, the value of the constant K_0 becomes

$$K_0 = -[(u_0H_x - H_v)^2 - Q(I_y + J_y)(u_0{}^2 - \tfrac{1}{2})]/(1 - u_0{}^2) \tag{34}$$

This value of K_0 introduced into Eq. (31) gives

$$F(u) = -[(uH_x - H_v)^2 + Q(I_x + J_y)(u^2 - \tfrac{1}{2})] + [(u_0H_x - H_v)^2 + Q(I_x + J_y)(u_0{}^2 - \tfrac{1}{2})] \times (1 - u^2)/(1 - u_0{}^2) \tag{35}$$

Combining Eqs. (35, 25, and 32) gives the description of the motion under the stated initial boundary conditions. If there is a restriction to cases in which the deviations of the satellite attitude from the nominal are small, the equations can be simplified. For the case $\delta_0 \approx 90°$, neglecting second-order terms, $\cos^2\delta_0 = u_0{}^2 \approx 0$, and Eq. (35) becomes

$$F(u) = -[H_x{}^2 + H_v{}^2 + Q(I_x + J_y)]u^2 + 2H_xH_v(u - u_0) \tag{36}$$

Combining Eqs. (36) and (32) and integrating gives

$$t_1 = \frac{(I_y + J_y)}{[H_x{}^2 + H_v{}^2 + Q(I_x + J_y)]^{1/2}} \times \text{arc sin} \times \left[\frac{2[H_x{}^2 + H_v{}^2 + Q(I_x + J_y)]u - 2H_xH_vu_0}{\{4H_x{}^2H_v{}^2 - 8H_xH_vu_0[H_x{}^2 + H_v{}^2 + Q(I_x + J_y)]\}^{1/2}}\right]_{u_0}^{u} \tag{37}$$

Defining Ω and A as follows:

$$\Omega = [H_x^2 + H_v^2 + Q(I_y + J_y)]^{1/2}(I_x + I_y)^{-1}$$

$$A^2 \equiv \frac{H_x^2 H_v^2 - 2H_x H_v u_0[H_x^2 + H_v^2 + Q(I_x + J_y)]}{[H_x^2 + H_v^2 + Q(I_x + J_y)^2]^2} \qquad (38)$$

we obtain from Eq. (37) $A \sin(\Omega t) = (u - u_0)$. For the case of practical interest where $\omega_s \gg \omega_0$

$$Q(I_y + J_y) \ll H_x^2$$

and Eq. (38) simplifies to

$$A^2 = H_x^2 H_v^2/(H_x^2 + H_v^2)^2 - [2H_x H_v/(H_x^2 + H_v^2)]u_0 \qquad (39)$$

These equations show that the boom tip vibrates in a horizontal plane with angular frequency Ω and amplitude LA, where L is the length of the boom. If vibration is measured for a unit length boom the amplitude is A. The vibration amplitude is measured from the initial position u_0, and Ω is called the nutation frequency. A is a constant because of Eq. (16).

The motion of the boom around the local vertical given by Eq. (20) is not further investigated here because the primary interest in an attitude-stabilized satellite is in u, which gives the deviation of the boom from the local vertical.

For the special initial condition with the boom in the vertical position, ($\delta_0 = 90°$), $u_0 = 0$, and from Eq. (39), $A = H_x H_v/(H_x^2 + H_v^2)$. The complete equation for the motion becomes

$$u = H_x H_v (H_x^2 + H_v^2)^{-1} \sin[(H_x^2 + H_v^2)^{1/2}(I_y + I_z)^{-1}t] \qquad (40)$$

For this special case, $H_z = H_v$.

It has been shown that the angular nutation frequency

$$\Omega = (H_x^2 + H_v^2)^{1/2}/(I_y + J_y) \text{ and because } H_x > H_v$$
$$\Omega \approx H_x/(I_y + J_y) \qquad (41)$$

is a constant determined by the satellite design and spin speed [Eq. (1)]. Therefore, if one desires to damp out the satellite vibrations, one needs only to add a nutation damper tuned to the frequency Ω. For high spin velocities, H_x is large, and Ω is a high frequency and therefore can be damped out easily in a short time.

The nutation damper acts also as a precession damper (Fig. 5). Such a damper aligns the spin axis of the satellite with the axis of the total angular momentum (**H** axis of Fig. 5), thus eliminating H_y and H_z and making H_x identical with **H**. Such damping devices have been demonstrated successfully in spin-stabilized satellites such as Telstar[4] and Syncom. Telstar uses a tungsten ball rolling inside a gas-filled, curved, closed tube. Syncom uses a closed tube partially filled with mercury. The action of such a device is shown in Fig. 7. In Fig. 7a, the closed tube with mercury is shown on the top. The orbital velocity $\boldsymbol{\omega}_0$ is not in line with the spin velocity $\boldsymbol{\omega}_x$. Therefore, the resulting angular velocity vector $\boldsymbol{\omega}$ is inclined with respect to the spin axis of the satellite. This causes the mercury (shown cross-hatched) to flow to the left of the tube because of the existing centrifugal force. In Fig. 7b, the same satellite is shown one-half revolution later, and the mercury is on the right-hand side because of the centrifugal force. Thus, the mercury goes back and forth once per revolution and causes a damping moment in line with the angular velocity vector until the spin axis is aligned with the total velocity vector as shown in Fig. 7c. In this condition, the mercury settles at the location farthest away from the axis.

A similar effect is achieved by the eddy currents induced in the satellite by the earth's magnetic field. The moment vector due to these currents is parallel to the total angular velocity vector; this has the effect of aligning the spin axis (x axis) of the satellite with this total angular velocity vector, thus eliminating precession and nutation. If this spin vector (x axis) forms with the local vertical an angle δ that is different from 90°, then according to Eq. (38) the gravity-gradient field acting on the boom causes continued precession and nutation that again causes misalignment between the total angular momentum vector and the spin axis. Thus, the damping device will again align the spin axis with the new total angular momentum vector.

The action of the damping device and of the gravity gradient, which have been described separately, will take place simultaneously and cause angle δ gradually to increase until it has reached 90°. After that, the gravity gradient is decoupled from the spinning satellite.

The effect of a misalignment in yaw (angle ψ) remains to be discussed. As a result of the coupling of roll and yaw by the orbital rotation [Eq. (19)], a yaw misalignment will become a roll misalignment one-quarter-orbit period later. In other words, angle ψ is converted to $(90 - \delta)$ after one-quarter-orbit period. It has been shown how the combined action of the gravity gradient and the damper maintains $\delta \approx 90°$. The yaw misalignment ($\psi \neq 0$), therefore, will be stabilized by the same action one-quarter period later. Therefore, the simplifying assumption that $\psi \simeq 0$ is justified as an approximation for the proper initial conditions.

Satellite Under Disturbing Torques

The disturbing torques that this attitude control system can handle are due primarily to solar radiation pressure and will be discussed here. (Other disturbing torques may arise from magnetic effects, micrometeorites, etc. This discussion still is restricted to circular polar or equatorial orbits.) The torque exerted by the boom should exceed the largest disturbing torque in order to assure attitude stability under all conditions. This is easily achieved with booms of reasonable length.[3]

In polar orbits, the sun is essentially stationary during a one-orbit period. Consider a polar orbit where the sun is in the orbital plane. In an attitude-stabilized satellite, the sun illuminates one-half of the satellite while the satellite travels from the equator over the North Pole to the equator and the other half while the satellite travels from the equator over the South Pole back to the equator. If the unbalanced

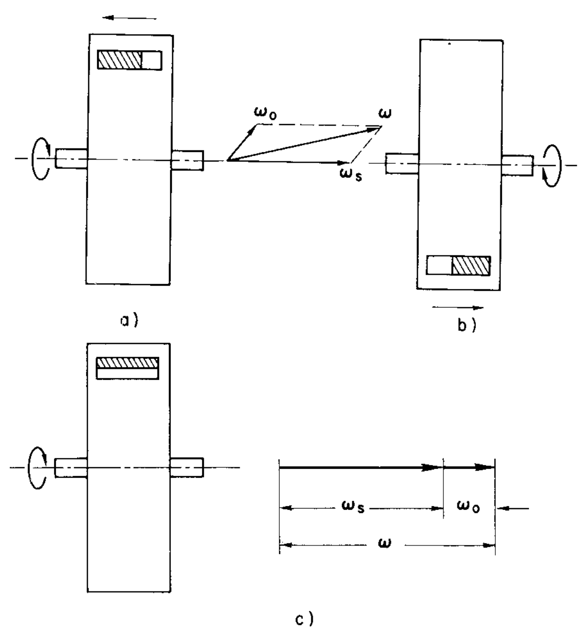

Fig. 7 Precession damper.

solar radiation pressure torques of the two halves are equal and opposite, no secular torque term arises from the solar radiation. If they are unequal, a secular torque term will arise. Of course, one-quarter year later the sun will be normal to the orbital plane. Under these circumstances, moments due to solar radiation are periodic with the orbit period.

In equatorial orbits, the sun rotates around the satellite with a period corresponding to the difference between the satellite period and 24 hr. In general, this periodic variation of the direction of solar radiation should not give rise to secular terms; however, there are special cases when secular terms can also arise in equatorial orbits. For instance, when the satellite precession period is of the same order of magnitude as the solar period, it is possible for a resonance between the two periods to cause a secular transfer of momentum from the solar radiation to the satellite.

Secular angular momentums can be minimized by proper design of the satellite. The secular terms can cause damping, as do, for instance, eddy currents induced by the earth's magnetic field. There is also a possibility that unbalances due to solar radiation pressure can cause an excitation of resonant vibrations or addition of secular angular momentums. Therefore, it is pertinent to investigate the satellite behavior under these terms. It must be noted, however, that because of the rotation of the earth around the sun, secular terms are really periodic terms of a one-year period. Therefore, we have in reality only short-period and long-period terms.

Secular angular momentum added to the spin axis will be due primarily to eddy currents induced by the magnetic field of the earth and will lead to a decay in spin velocity, but this can be corrected easily by the jet and deflection vane shown in Fig. 4.

Secular angular momentum added to the pitch axis can be handled easily by the gravity-gradient-actuated pitch axis control system, described previously, provided that the system is designed so that the torque of the boom is larger than the secular torque. Secular angular momentum added to the y axis can be damped readily by the nutation damper (as referenced in Fig. 7).

Secular torque about the vertical axis $\mathbf{H}_v$, which has a component in roll (y) and yaw (z) (Fig. 5), can be damped by the precession damper described. Furthermore, 1) a perfectly symmetric satellite has no torque due to solar radiation pressure, 2) asymmetrics can give rise to a torque that is generally periodic with the orbit period, and 3) these torques can have secular terms (really one-year period) if they are not equal and opposite (second-order effect).

Summary and Conclusions

The dynamics of the satellite combining spin stabilization with gravity-gradient stabilization result in a stable system with a fixed nutation frequency that can be made high by design. Vibrations in the roll and yaw axes are easily damped by a nutation damper in the spinning part.

The system described is basically passive as far as the control torques are concerned; however, active means are used to damp pitch axis vibrations of the stationary part. They are damped by a single cold-gas pulsed jet that also serves for station-keeping and compensation of spin rate decay. Thus, this system can achieve high accuracy of stabilization with little oscillatory perturbation, in addition to the advantages of high antenna gain.

With a high spin angular momentum, secular momentum terms about the vertical (yaw or precession axis) have no adverse effect on the system. Considering that the so-called secular terms are really terms of a one-year period, they do not limit the lifetime of the system.

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